Hand calculation of PI to 20 digits

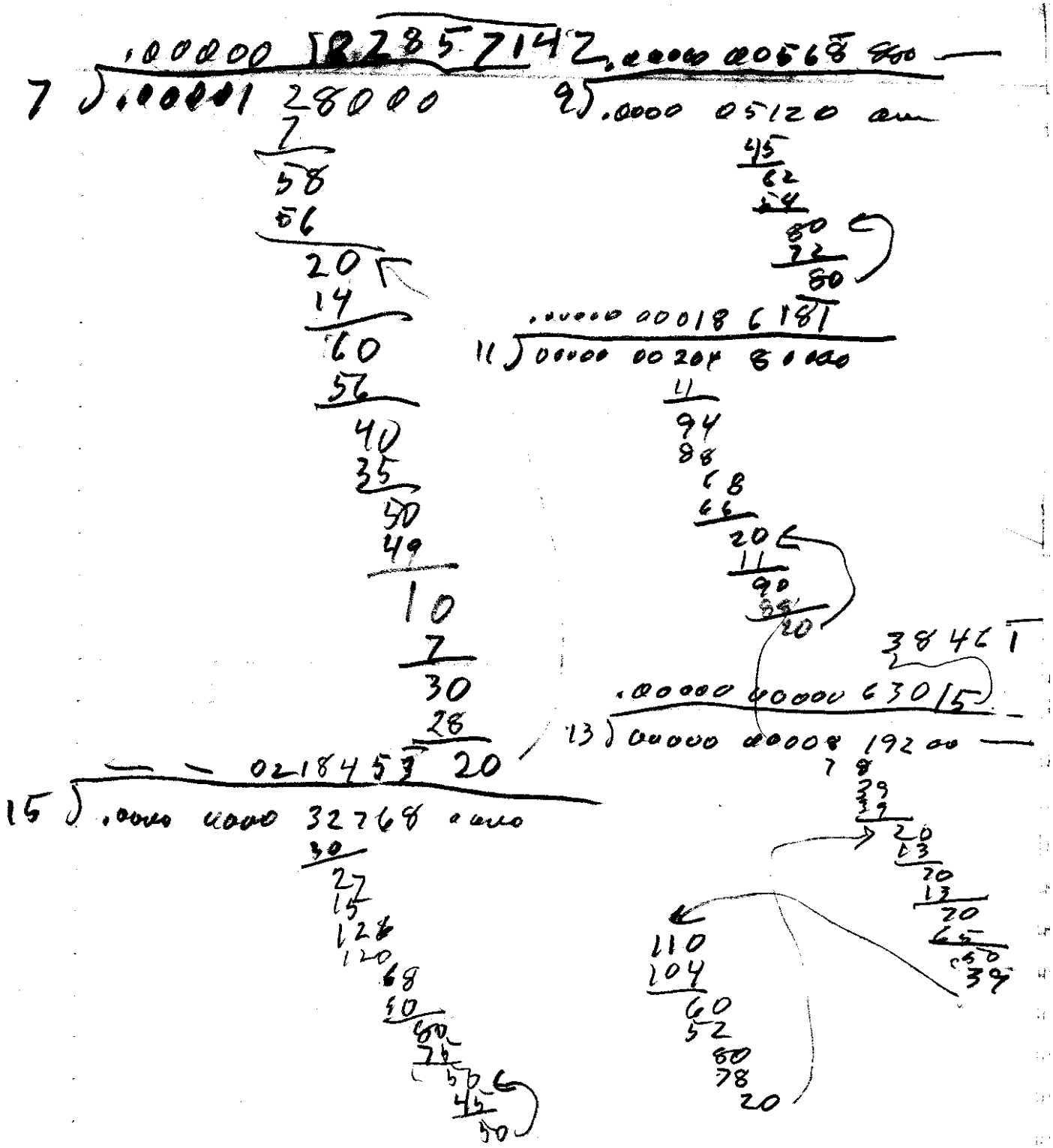
The following seven pages contain the hand calculation of PI to 20 digits of which there was an error of 14 units in the last digit, affected the last two digits and resulted in 18 correct digits. This is more than digits needed for most every day needs. The planet Pluto is on the average 6,000,000,000 km from the sun, this would be 6X10^15 mm, so this value of PI would give the circumference the nearest mm. This example has several problems first we do not know the distance to the mm and the orbit is not a circle or a perfect ellipse.

In 1706 John Machin discovered the following formula relation and used it to calculate 100 digits of PI. This formula has become one of the most used since it was discovered, PI = 4 * (4 * ATN(1/5) - ATN(1/239). The function ATN(X) = X - X^3/3 + X^5/5 – X^7/7 + X^9/9 etc. this relation can be used to calculated the ATN function. -

The first three pages are for the ATN(1/5). The forth page is for has the final calculations to produce the final value of PI. The last three are the calculation of ATN(1/239). In the left margin I produced a table of values 57121 and as a check the last value is 10 times so I would know that there were no errors.

I started the hand calculation doing 10 digits and then expanded it to this 20 digit example, which rook about 2 hours. To do the full 40 digit would take several part days and 100 digits took 23 part days. I have started to do the work on 40 digits with 4 10 digits group and I have found it to hard to keep the columns straight, I will have to start over with 5 digit groups. To equal the work of John Machin would take several weeks and a whole lot more paper. To do 100 digits the 1/5 has 70 terms while the 1/239 has 21 terms and each term has 5 times as many digits as the example here. . For more of my work on PI go to http://engert.us/erwin/Miscellaneous.html where you can find this and other files.

1	.20000	00000	00000	00000
3	.00800	00000	00000	00000
5	.00032	00000	00000	00000
7	.00001	28000	00000	00000
9	.00000	05120	00000	00000
11	.00000	00204	80000	00000
13	.00000	00008	19200	00000
15	.00000	00000	32768	00000
17	.00000	00000	01310	72000
19	.00000	00000	00052	42880
21	.00000	00000	00002	09712
23	.00000	00000	00000	08388
25	.00000	00000	00000	00335
3	.00266	16664	12223	36663
7	.00000	18285	71428	57142
11	.00000	00016	61818	18181
15	.00006	00000	02184	53333
19	.00000	00000	00002	75941
23	00000	00000	00000	00364
−	.00266	84971	02100	71627
1	.20000	00000	00000	00000
5	.00006	40006	00000	00000
9	.00060	00568	88889	88888
13	00000	00000	63015	38461
17	00000	00000	00077	10117
21	00000	00001	00000	09986
25	00000	00000	00000	00013
	.20007	40569	51991	47465
−	00266	84971	02100	71627
ATN(1/5)	19739	55598	49880	75838



$$
\begin{array}{r}
-00077\ 10117 \\
17\overline{)-01310\ 72000} \\
119 \\
\hline
120 \\
119 \\
\hline
17 \\
17 \\
\hline
20 \\
17 \\
\hline
30 \\
17 \\
\hline
130 \\
119 \\
\hline
11
\end{array}
$$

$$
\begin{array}{r}
-00002\ 75941 \\
19\overline{)-00052\ 42880} \\
38 \\
\hline
144 \\
133 \\
\hline
112 \\
95 \\
\hline
178 \\
171 \\
\hline
78 \\
72 \\
\hline
20 \\
17 \\
\hline
3
\end{array}
$$

$$
\begin{array}{r}
09986 \\
21\overline{)\ 2\ 09715} \\
189 \\
\hline
207 \\
189 \\
\hline
181 \\
168 \\
\hline
135 \\
126 \\
\hline
9
\end{array}
$$

$$
\begin{array}{r}
00364 \\
23\overline{)\ 08388} \\
69 \\
\hline
148 \\
138 \\
\hline
108 \\
92 \\
\hline
16
\end{array}
$$

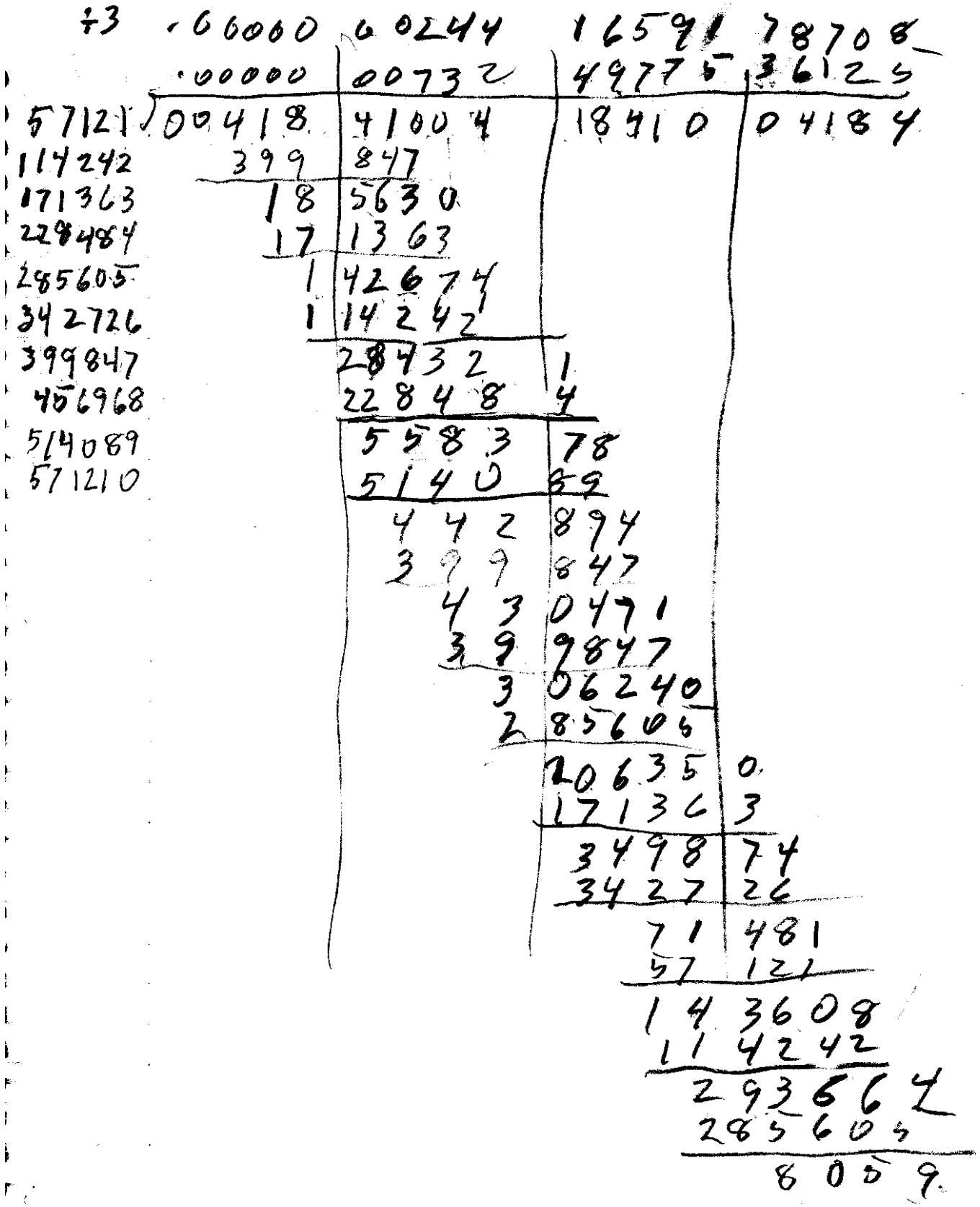
3	0 0 0 0 0	0 0 2 4 4	1 6 5 9 1	7 8 7 0 8
7				0 0 3 2 0
	0 0 0 0 0	0 0 2 4 4	1 6 5 9 1	7 9 0 2 8
1	. 0 0 4 1 8	4 1 0 0 4	1 8 4 1 0	0 4 1 8 4
5			0 0 2 5 6	4 7 2 3 1
+	. 0 0 4 1 8	4 1 0 0 4	1 8 6 6 6	5 1 4 1 5
−		0 0 2 4 4	1 6 5 9 1	7 9 0 2 8
ATN($\frac{1}{239}$)	0 0 4 1 8	4 0 7 6 0	0 2 0 7 4	7 2 3 8 7

1	0 0 4 1 8	4 1 0 0 4	1 8 4 1 0	0 4 1 8 4
3	0 0 0 0 0	0 0 7 3 2	4 9 7 7 5	3 6 1 2 5
5	0 0 0 0 0	0 0 0 0 0	0 1 2 8 2	3 6 1 5 7
7	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0	0 2 2 4 4

$ATN(\frac{1}{5})$	.19739	55598	49880	75838
$\times 4$				$\times 4$
$4 \times ATN(\frac{1}{5})$	.78958	22393	99523	03352
$-ATN(\frac{1}{239})$	.00418	40760	02074	72387
$\pi/4$	.78539	81633	97448	30965
$\times 4$	.			$\times 4$
π	3.14159	26535	89793	23860

$$
\begin{array}{r}
.00418\ 41 \\
239\overline{)1.00000\ \ 00000\ \ 00000\ \ 00000} \\
\quad 956 \\
\hline
\quad 440 \\
\quad 239 \\
\hline
\quad 2010 \\
\quad 1912 \\
\hline
\quad\quad 980 \\
\quad\quad 956 \\
\hline
\quad\quad\quad 240 \\
\quad\quad\quad\ \ 1
\end{array}
$$

$$
\begin{array}{r}
239 \\
\times\ 239 \\
\hline
2151 \\
717 \\
478 \\
\hline
57121
\end{array}
$$



÷6

00256 472314

00000 01282 36157

57121) 00000 00732 49775 36125

571 71

161 287

114 242

47 0457

45 6968

1 34895

1 14242

-- 000.00 02244 20653 3

57121) -- 01282 36125 17136 3

114242 351.706

139971 34272 6

114342 89801

255992 57121

228484 326802

27508 285609

22848 41975

4660 1 39847

12128

0320

7) 2244